ANALYSIS OF POWER TRANSFORMERS UNDER TRANSIENT CONDITIONS

by

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ABSTRACT

Low specific weight transformers may be designed to operate under impulse conditions well above the steady state limit. This usually results in nonlinear current and voltage transformation. A procedure is outlined in this paper for the analysis of lumped parameter transformer models with nonlinear self-inductance. An algorithm for modeling inductance is developed which is accurate for square loop cores as well as ordinary soft magnetic materials. A simple routine which can be implemented with modest computing power is used to determine the dynamic response of transformers driven by a variety of sources. The model permits independent assignment of initial conditions for the magnetic state of the core and the phase of the driving source. This permits a computation of inrush currents and output waveforms under the entire range of possible initial conditions. This work was sponsored by the United States Air Force Aero Propulsion Laboratory, Wright Patterson Air Force Base, Ohio.

Introduction

The problem of analysis of the transient behavior of power transformers breaks down to two major questions. First, can a lumped parameter model adequately represent the behavior of such a device and second can a model of the nonlinear self-inductance be devised which has sufficient accuracy and flexibility to be useful? In this paper, the validity of a lumped parameter model is assumed and the answer to the second question is the subject of discussion. There is evidence [2], that prior to the development of the present model the answer may have been negative. Since all of the other circuit elements are linear, a Thevenin equivalent for the network can always be found. Therefore a particularly simple equivalent circuit is used to place emphasis on the magnetic model. The resulting circuit equation is both discontinuous and nonlinear. Thus requiring numerical solution with special provisions for handling the discontinuities.

A simplified lumped parameter equivalent circuit is shown in Fig.1 where R_1 is the equivalent resistance, L_1 the leakage inductance, L_2 the self-inductance C the shunt capacitance and R_2 the parallel combination of the

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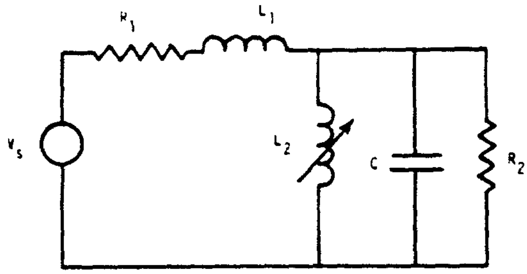


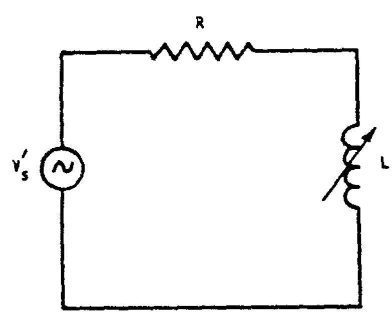
Simplified transformer equivalent circuit

Fig. 1

reflected load resistance and the effective core loss term. All of the components are linear with exception of L_2 in this model. Ignoring L_1 and C permits the circuit to be transformed to that shown in Fig.2 where

$$V_s' = V_s \frac{R_1}{(R_1 + R_2)} \tag{1}$$

$$R = \frac{R_1 R_2}{(R_1 + R_2)} \tag{2}$$



Typical L-R circi with nonlinear inductance

3. 2

The voltage dropped acrc the nonlinear inductance L_2 is computed by applying Snell's law to the B-H curve model described in Ref. 1. (This model is discussed in Appendix A.) The resulting modeled permeability is

$$\mu = \mu_o + \frac{B_s H_c \left(\frac{B_s}{B_r} - 1\right)}{\left[H_c\left(\frac{B_s}{B_r} - 1\right) + |H + KH_c|\right]^2} \qquad (3)$$

and the resulting inductive voltage drop is

$$V_L = \frac{N^2 A_c}{\mathcal{L}} \mu \frac{dI}{dt} = L(I) \frac{dI}{dt} \qquad (4)$$

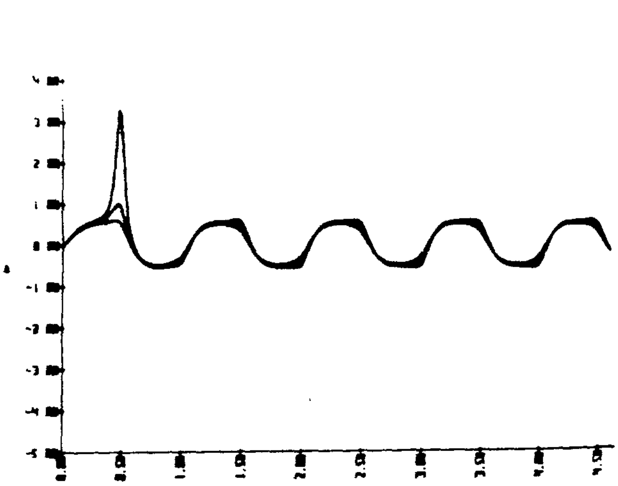
The symbols in equations (1) through (4) are identified in Appendix B. Combining equations (1) through (4) with the remaining circuit voltages in Fig. (2) yields

$$\frac{dI}{dt} = \left(\frac{R_1}{R_2 + R_1}\right)\left(V_s - R_2 I\right) / L(I) \qquad (5)$$

This differential equation could be solved by a number of standard methods were it not for the dis continuities which occur at the tips of the hysteresis curves. The method of isoclines has been adapted to this problem in a manner which is stable under most conditions. This technique was implemented on an Hp 9830 machine using their modified Basic language. Returning now to Fig. (1), it may be shown that this circuit is reducible to the form of Fig. (2) with the result that the resistance and source become complex. Furthermore, the poles and zeros of the equivalent voltage and impedance terms may be used to obtain the break frequencies for any given set of parameter values. A specific example was chosen for which the break frequencies were well away from the source frequency. So the series impedance could be assumed to be totally resistive. The parameter values were

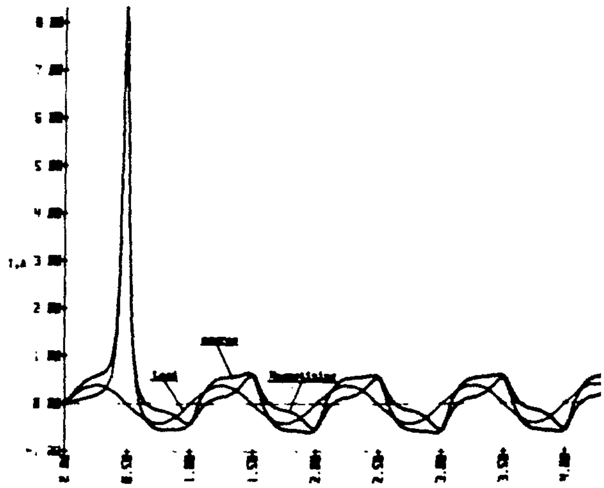
$N = 100$	$A_c = 4.3\ E\text{-}06\ m^2$
$\mathcal{L} = 8.47\ E\text{-}02\ m$	$B_s = 1.8\ T$
$B_r = 1.0\ T$	$H_c = 40\ At/m$

Fig. (3) is a plot of the source current for a four volt sinusoidal source whose frequency is 1 kHz for R_1 equal to 0.1, 0.5, and 1.0 ohms, and $R_2 = 10$ ohms. For this case as well as others in which a small core was used at frequencies for which the other reactances could be ignored, the inrush component in the current died out in about one cycle. Fig. (4) shows load, source and magnetizing currents for the same core. Again the inrush transient is seen to disappear after one cycle.



Source Current For a Lumped Parameter Transformer Model

Fig. 3



Currents a Lumped Parameter Transformer Model

Fig. 4

Concluding Remarks

This paper contains an outline of a method which is presently being used to analyze the behavior of steady state power transformer designs under transient conditions. It is based on a simple lumped parameter model of the transformer with a new emperical expression for the inductance. The results of this technique applied to small sample transformers indicates that for cases where the frequency is low enough to ignore leakage reactance and capacitive effects, the worst case inrush transient will die out in about one cycle. The case for larger transformers and higher frequencies have yet to be studied, but the present results provide an encouraging indication that the technique is successful.

References

(1) "Final Technical Report on Development of Lightweight Transformers for Airborne High Power Supplies" AFAPL -TR-75 15, June, 1975

(2) Wm. A. Manly Jr. " An Appraisal of Several Nonlinear Hysteresis Loop Models" IEEE Trans. on Magnetics, Vol. MAG-9, No.3, Sept., 1973

Apper .x A

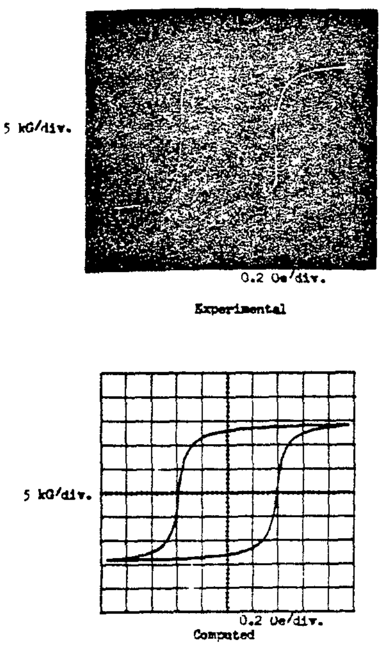
Magnetic Properties Modeling

The model is generat from three experimentally determined parameters. T .se are the coercive force, the saturation flux density, and the residual flux density. The mathematical expressions for B(H) are shown in equations A1 and A2 respectively.

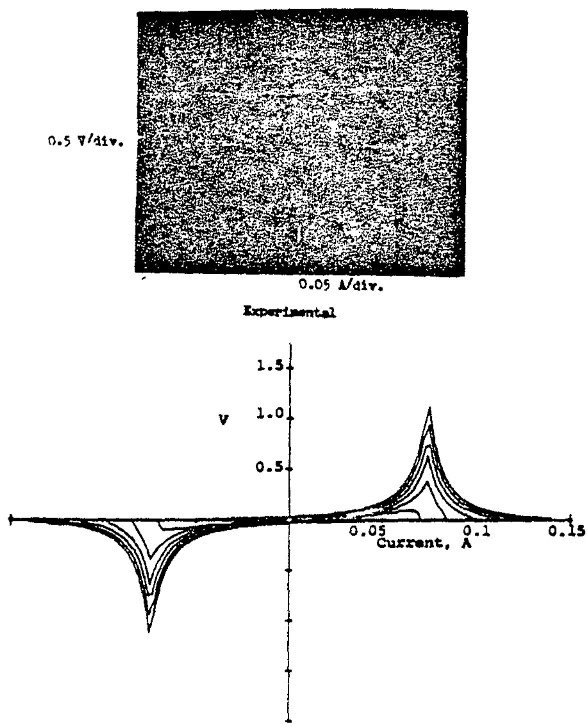
$$B = \frac{B_s\left(H + KH_c\right)}{H_c\left(\frac{B_s}{B_r} - 1\right) + \left|H + KH_c\right|} - KB_D \qquad \text{(A1)}$$

$$B_D = 1/2\left[\frac{B_s\left(H_m + H_c\right)}{H_c\left(\frac{B_s}{B_r} - 1\right) + \left|H_m + H_c\right|} - \frac{B_s\left(H_m - H_c\right)}{H_c\left(\frac{B_s}{B_r} - 1\right) + \left|H_m - H_c\right|}\right] \quad (A2)$$

Fig. (A-1) shows experimental and computed hysteresis curves for 3% nickel 97% iron samples. It was pointed out by Manly 2 that this type of data is difficult to compare quantitatively. Using a reduced pulse width from the current-voltage characteristics he showed that none of his five models could fit the data with any precision. Using the reduced pulse width from the computed I-V characteristics shown in Fig. (A-2) it is found that the values for this model fall precisely in the region appropriate to tape wound cores.



Experimental and Computed Hysteresis Curves
Fig. (A-1)



I-V Characteristics

Fig. (A-2)

App lix B

Symbol List

A_c	- Core Cross-sectional Area	K	- ±1 (Pos. for upper curve)
B	- Flux Density	L	- Inductance
B_d	- Displacement Flux Density	$\mathcal{L}$	- Magnetic Path Length
B_r	- Residual Flux Density	N	- Number of Turns
B_s	- Saturation Flux Density	R_1	- Series Resistance
H	- Magnetizing Force	R_2	- Shunt Resistance
H_c	- Coercive Force	R	- Equivalent Resistance
H_m	- Maximum Magnetizing force	t	- Time
I	Current	V_s	- Source Voltage
μ	- Permeability	V_s'	- Equivalent Source Voltage